Balanced Search Trees

- Resources available:
  kevin-miao.com

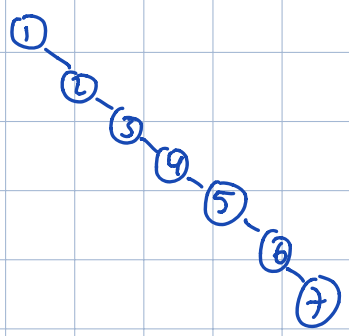
Today

- Quiz #2: Review
- Mini Review: Balanced Search Trees
  - → 2-3-4 Trees
  - → LLRB
    - * Rotations
    - * Insertion

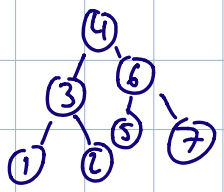
Mini Review

- Why balanced trees?
  Efficiency at all times

add 1, 2, 3, 4, 5, 6, 7



vs



O(log(n))

* 2-3-4 trees

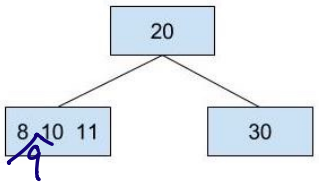
  * Can have 2, 3 or 4 children

Insertion * Go down like BST, add to leaf

* If invalid tree, pop up middle

repeat until valid

valid is when for all nodes, the items ≤ 3



2-3-4 tree

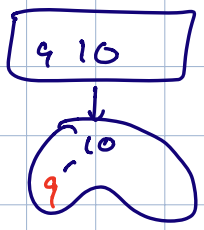
≤1 1-2 2-3 ≥3

Variants (B-Trees)

→ 2-4

→ 2-3-4-5

→ 2-3 Trees



Red Black trees ⟷ 2-3-4 Trees

* Middle Element ⟷ black node reachable from parent

* left ⟷ red left of black

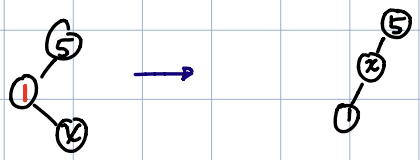
* Right ⟷ red right of black

Left leaning Red Black Trees ⟷ 2-3 Trees

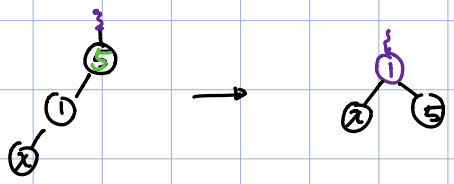
* Left → Red : Left of black

Operations

① Rotate left (1)

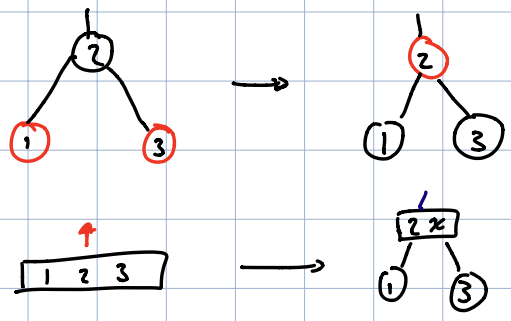


② Rotate Right (5)



③ Flip Color

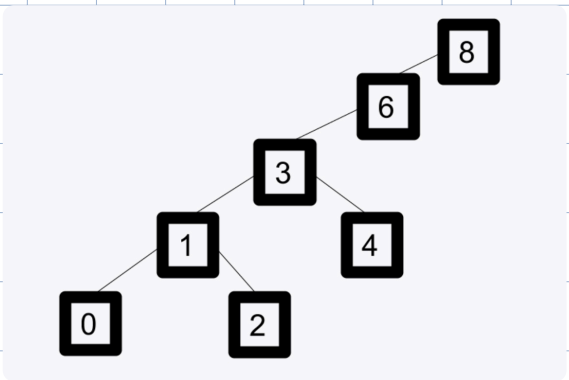
(only LLRB)



Quiz

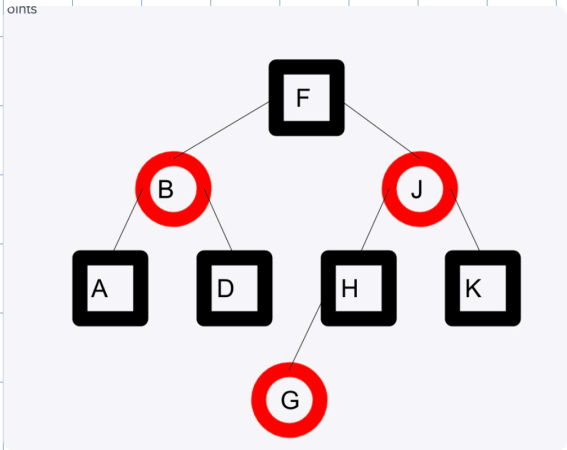
Q3

a)



Which sequence of rotations will balance this tree? (Height = 3)

b)



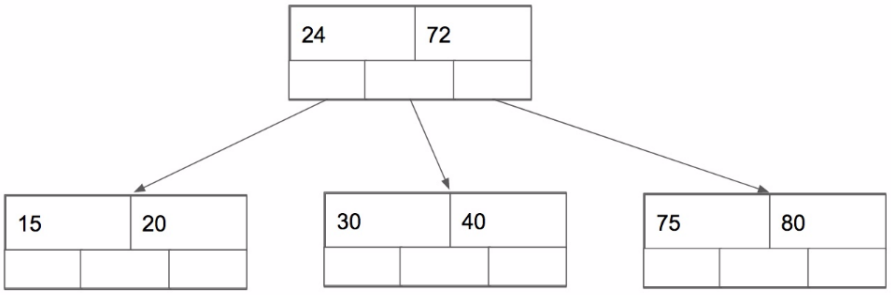
Convert this tree to a 2-3-4 tree.

4 All about Trees

1. Why does a binary search tree have a worst case runtime of $\theta(n)$ for *contains*?

2. Give a sequence of operations, such that if they were inserted in the order they appear, would result in a "poor" binary search tree.

3. Examine this B-tree with order 3. Mark the paths taken when the user calls *contains*(40).



4. Now call *insert*(35), and draw the resulting tree.

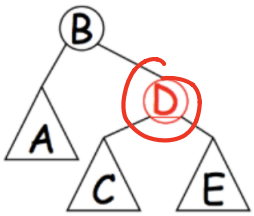
5. What property of a B-tree rectifies problems of binary search trees, such as the one in 1.1? Why would you not use a B-tree?

5 The Holy LLRB Invariant

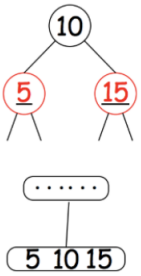
RB Tree Invariants: Node labels are in order from left to right. All paths through the tree contain the same number of black nodes. No red nodes have red parents. As a result, the height of a RB tree with n nodes is $O(logn)$.

LLRB trees must also maintain the following invariant (in addition to the regular red-black invariant):

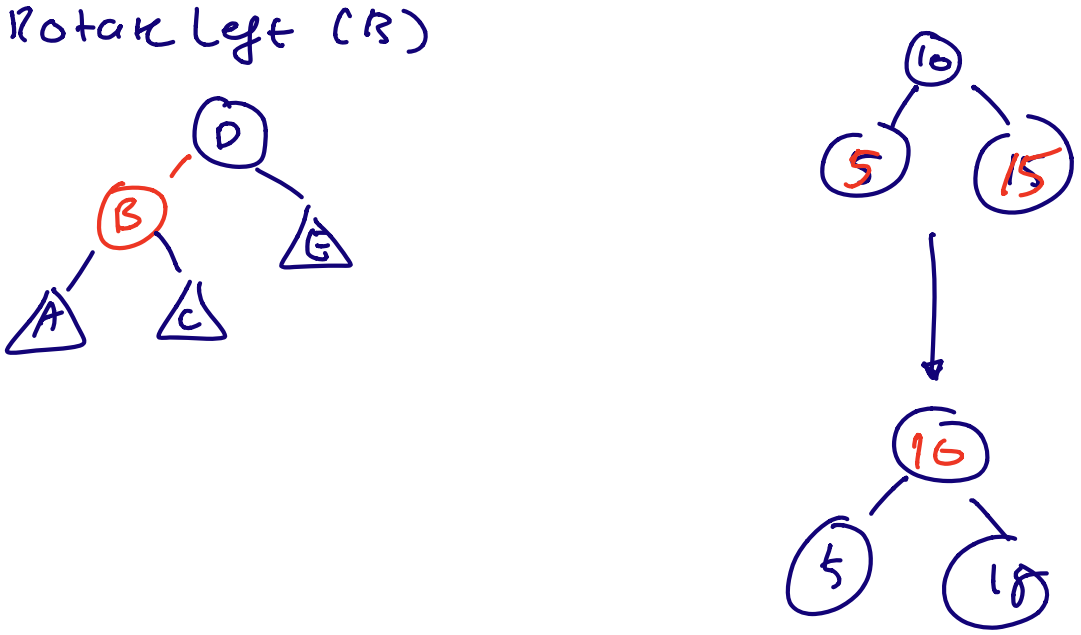
No right-leaning trees (black parent with right red child):



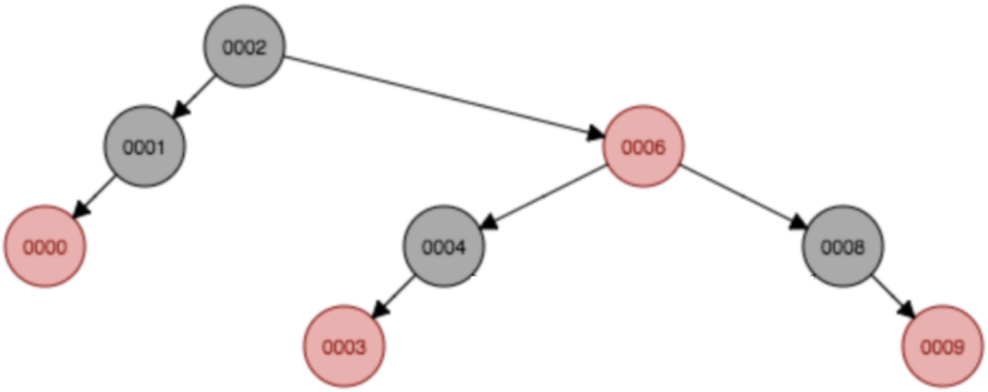
No "4-nodes" (black parent with two red children):



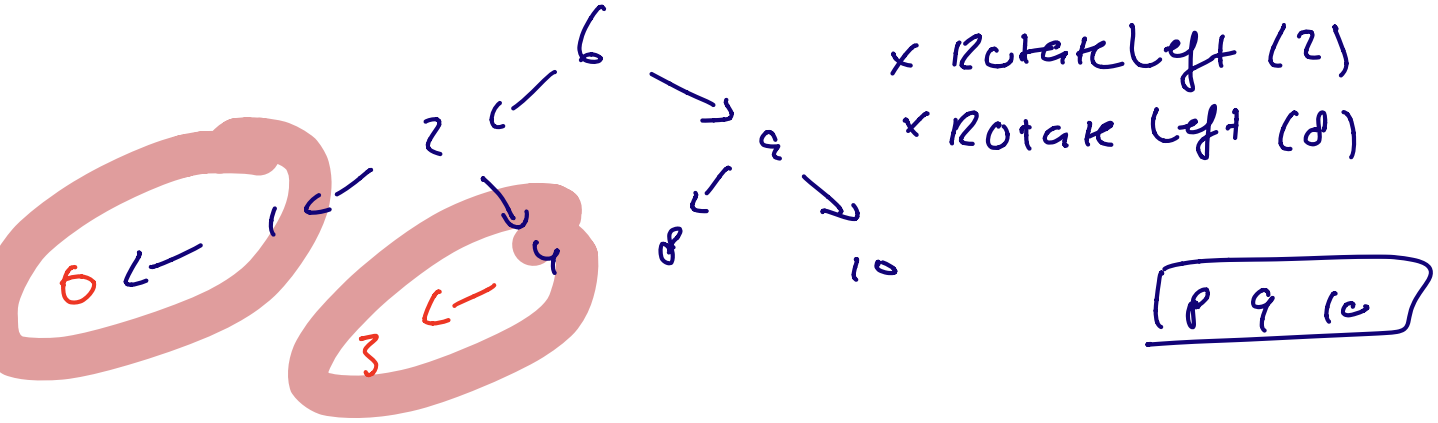
1. What are the "fixups" for the two cases above in order to preserve the LLRB invariant (i.e. what operations do we perform on each tree to ensure it is a proper LLRB)?



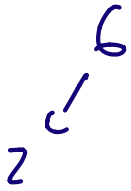
Consider the following RB tree:



2. Draw the tree after applying all necessary fixups to make it a proper LLRB tree.



3. Next, insert 10 into the tree, and apply all fixups to preserve the LLRB invariant.



4. Finally, draw the corresponding 2-3 tree.